

Relaxation of the dynamical gluino phase and unambiguous electric dipole moments

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We propose a new axionic solution of the strong CP problem with a Peccei-Quinn mechanism using the gluino rather than quarks. The spontaneous breaking of this new global $U(1)$ at 10^{11} GeV is related to the supersymmetry breaking scale of 1 TeV (replacing the μ parameter with a composite field operator) and results in the MSSM (minimal supersymmetric standard model) with R parity conservation. In this framework, electric dipole moments become calculable without ambiguity.

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CP nonconservation is a fundamental issue in particle physics. We know that CP is not conserved in K - \bar{K} mixing (i.e., $\epsilon \neq 0$) [1] and in K decay (i.e., $\epsilon' \neq 0$) [2]. However, only an upper limit ($0.63 \times 10^{-25} e \text{ cm}$) exists for the electric dipole moment (EDM) of the neutron [3]. This may not be so bothersome until we realize that the currently accepted theory of strong interactions, i.e., quantum chromodynamics (QCD), actually violates CP through the instanton-induced term [4]

$$\mathcal{L}_\theta = \theta_{QCD} \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} G_a^{\mu\nu} G_a^{\alpha\beta}, \quad (1)$$

where g_s is the strong coupling constant, and

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s f_{abc} G_b^\mu G_c^\nu \quad (2)$$

is the gluonic field strength. The value of θ_{QCD} must then be less than 10^{-10} in magnitude (instead of the expected order of unity) to account for [5] the nonobservation of the neutron EDM. This is known as the strong CP problem.

Another fundamental issue in particle physics is supersymmetry. It allows us to solve the hierarchy problem so that our effective theory at the electroweak energy scale (M_W) is protected against large radiative corrections. However, this requires the scale of soft supersymmetry breaking (M_{SUSY}) to be not much higher than M_W .

In the following we address the question of how the strong CP problem is to be solved in the context of supersymmetry. We find that it can be achieved with a new kind of axion [6,7] which couples to the gluino rather than to quarks. In a natural implementation of this idea, we find that the breaking of supersymmetry must be related to the axion, with the μ parameter replaced by a composite field operator containing the axion.

In our framework, the θ_{QCD} contribution to any quark EDM is canceled exactly by the minimization of the dynamical gluino phase. Hence the calculation of EDM's in the minimal supersymmetric standard model (MSSM) becomes unambiguous. The possibility of cancellation

[8] among other different CP nonconserving contributions [9] to EDM's can now be pursued without fear of contradiction.

With the addition of colored fermions, the parameter θ_{QCD} of Eq. (1) is replaced by

$$\bar{\theta} = \theta_{QCD} - \text{Arg Det } M_u M_d - 3 \text{ Arg } M_{\tilde{g}}, \quad (3)$$

where M_u and M_d are the respective mass matrices of the charge $=2/3$ and charge $=-1/3$ quarks, and $M_{\tilde{g}}$ is the mass of the gluino. The famous Peccei-Quinn solution [6] is to introduce a dynamical phase to the quark masses which then relaxes $\bar{\theta}$ to zero. The specific realization of this requires an axion [7] which is ruled out experimentally [10]. Two other axionic solutions have been proposed which are at present consistent with all observations. The Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) solution [11] introduces a singlet scalar field as the source of the axion but its mixing with the doublet scalar fields which couple to the quarks is very much suppressed. The Kim-Shifman-Vainshtein-Zakharov (KSVZ) solution [12] introduces new heavy quarks so that the axion does not even couple directly to the usual quarks. Neither scheme requires supersymmetry.

In the context of supersymmetry however, it is clear that the *simplest* and *most natural* thing to do is to attach the axion to the gluino rather than to the quarks in Eq. (3). Because of the structure of supersymmetry, all other superparticles will be similarly affected. This is then a very strong hint that it may have something to do with the breaking of supersymmetry. As shown below with our proposed singlet complex scalar field S , whose phase contains the axion, certain soft supersymmetry breaking parameters (gaugino masses and A terms) are of order $|\langle S \rangle|^2 / M_{Pl}$, where $M_{Pl} \sim 10^{19}$ GeV is the Planck mass. Hence a value of 10^{11} GeV for $|\langle S \rangle|$, which is allowed by astrophysical and cosmological constraints, would imply $M_{SUSY} \sim 1$ TeV. Other soft supersymmetry breaking parameters (sfermion masses and B terms) are assumed to be also of order 1 TeV.

It is known [13] that a continuous global $U(1)_R$ symmetry [14] can be defined for the MSSM. The quark ($\hat{Q}, \hat{u}^c, \hat{d}^c$) and lepton (\hat{L}, \hat{e}^c) chiral superfields have $R = +1$ whereas

the Higgs (\hat{H}_u, \hat{H}_d) chiral superfields and the vector superfields have $R=0$. The superpotential

$$\hat{W} = \mu \hat{H}_u \cdot \hat{H}_d + h_u \hat{Q} \cdot \hat{H}_u \hat{u}^c + h_d \hat{Q} \cdot \hat{H}_d \hat{d}^c + h_e \hat{L} \cdot \hat{H}_d \hat{e}^c \quad (4)$$

has $R=+2$ except for the μ term (which has $R=0$). In the above, the Yukawa couplings $h_{u,d,e}$ are non-Hermitian matrices in flavor space. The resulting Lagrangian is then invariant only with respect to the usual discrete R parity, i.e.,

$$R \equiv (-1)^{3B+L+2J}, \quad (5)$$

where B is baryon number, L lepton number, and J spin angular momentum, hence R is even for particles and odd for superparticles.

We now propose to make $U(1)_R$ an exact global symmetry of the supersymmetric Lagrangian, as well as that of all the supersymmetric breaking terms. We introduce the composite operator

$$\mu(\hat{S}) \equiv \frac{1}{M_{Pl}} (\hat{S})^2, \quad (6)$$

where the singlet superfield \hat{S} has $R=+1$. Our model is then defined by the new superpotential

$$\begin{aligned} \hat{W} = & \mu(\hat{S}) \hat{H}_u \cdot \hat{H}_d + m_s^2 \mu(\hat{S}) + h_u \hat{Q} \cdot \hat{H}_u \hat{u}^c + h_d \hat{Q} \cdot \hat{H}_d \hat{d}^c \\ & + h_e \hat{L} \cdot \hat{H}_d \hat{e}^c, \end{aligned} \quad (7)$$

which has $R=+2$, thus yielding a supersymmetric Lagrangian which is invariant under $U(1)_R$. The so-called μ problem in the MSSM may then be solved if $|\langle S \rangle| \sim 10^{11}$ GeV (which is of course an assumption in itself), because the scale of $\mu(S)$ is $|\langle S \rangle|^2/M_{Pl}$ which is of order 1 TeV instead of the typical grand unification scale of 10^{16} GeV. We now add the following set of supersymmetry breaking terms which are also invariant under $U(1)_R$:

$$\begin{aligned} \Delta \mathcal{L} = & |\mu(S)|^2 [\tilde{Q}^\dagger Y_Q \tilde{Q} + \tilde{u}^{c\dagger} Y_u \tilde{u}^c + \tilde{d}^{c\dagger} Y_d \tilde{d}^c + \tilde{L}^\dagger Y_L \tilde{L} \\ & + \tilde{e}^{c\dagger} Y_e \tilde{e}^c] + \{\mu(S)^\dagger [k_u \tilde{Q} \cdot H_u \tilde{u}^c + k_d \tilde{Q} \cdot H_d \tilde{d}^c \\ & + k_e \tilde{L} \cdot H_d \tilde{e}^c] + \text{H.c.}\} + |\mu(S)|^2 [y_u |H_u|^2 + y_d |H_d|^2 \\ & + (k_\mu H_u \cdot H_d + \text{H.c.})] + \{\mu(S)^\dagger [k_3 \tilde{\lambda}_3^a \tilde{\lambda}_3^a + k_2 \tilde{\lambda}_2^i \tilde{\lambda}_2^i \\ & + k_1 \tilde{\lambda}_1 \tilde{\lambda}_1] + \text{H.c.}\}, \end{aligned} \quad (8)$$

where $\tilde{\lambda}_3^a$ is the gluino octet, $\tilde{\lambda}_2^i$ the $SU(2)_L$ gaugino triplet, and $\tilde{\lambda}_1$ the $U(1)_Y$ gaugino singlet. The parameters $k_{1,2,3}$ and k_μ are complex, whereas $y_{u,d}$ are real. The matrices $Y_{Q,L}$ and $Y_{u,d,e}$ are Hermitian, whereas $k_{u,d,e}$ are non-Hermitian. The terms proportional to $|\mu(S)|^2$ are of course invariant under $U(1)_R$ by themselves, so their coefficients are not necessarily of order $|\mu(S)|^2$. Here we must make the additional

assumption that the underlying mechanism of supersymmetry breaking is such that $\mu(S)$ acts as a kind of “messenger” field.

The Lagrangian $\Delta \mathcal{L}$ describes the interaction of $\mu(S)$ with sfermions, Higgs doublets, and gauginos only. However, a complete description of our model requires the self-interactions of the singlet to be specified as well. The pure singlet contribution $m_s^2 \mu(\hat{S})$ in \hat{W} is allowed by the symmetries of the model and the mass parameter m_s^2 is *a priori* arbitrary. Through the F -term contributions, this induces a positive mass-squared parameter for the singlet: $m_F^2 \equiv 4m_s^4/M_{Pl}^2$. However, interactions at higher energies at or near the Planck scale can provide an additional mass-squared parameter m_0^2 as well as a quartic coupling λ_s . Hence the effective potential for the singlet takes the form $V_s = M_s^2 |S|^2 + \lambda_s |S|^4$ with $M_s^2 \equiv m_0^2 + m_F^2$. Since the Higgs doublets have vanishing R charges, the electroweak breaking cannot have any effect on the fate of $U(1)_R$. The only way to break it is to allow the singlet to develop a nonvanishing vacuum expectation value. This can happen only when $M_s^2 < 0$ so that $v_s^2 = -M_s^2/2\lambda_s$. Since m_F^2 is positive, m_0^2 should be negative enough to induce a negative M_s^2 . This implies that m_s^2 cannot be as large as M_{Pl}^2 as it would leave $U(1)_R$ unbroken; hence $|m_s^2| \sim |m_0^2| \sim v_s^2$ is the optimal choice. The singlet field could then be expanded around v_s as

$$S(x) = \frac{1}{\sqrt{2}} [v_s + s(x)] e^{i\varphi(x)}, \quad (9)$$

where $\varphi(x)$ is the corresponding Nambu-Goldstone boson [15] which has a strictly flat potential, and $s(x)$ is a real scalar field with a mass of order v_s . It is clear from the above that our $U(1)_R$ plays the role of what is usually called $U(1)_{PQ}$ [6]. Whereas the conventional $U(1)_{PQ}$ applies to the usual quarks and leptons, our $U(1)_{PQ}$ applies only to the superparticles, and the gluino is the only colored fermion having a nonvanishing $U(1)_{PQ}$ current:

$$J_\mu^{5,\tilde{g}} = \bar{\lambda}^a \gamma_\mu \gamma_5 \lambda^a, \quad (10)$$

where we have used the four-component notation: $\lambda^a = (\tilde{\lambda}_3^a, \tilde{\lambda}_3^a)$. Now the gluino also contributes to the color current with respect to which $J_\mu^{5,\tilde{g}}$ has a nonvanishing quantum anomaly:

$$\partial^\mu J_\mu^{5,\tilde{g}} = \frac{6g_s^2}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} G_a^{\mu\nu} G_a^{\alpha\beta}. \quad (11)$$

Since $J_\mu^{5,\tilde{g}}$ couples to φ as $\partial^\mu \varphi J_\mu^{5,\tilde{g}}$, the effective QCD vacuum angle takes the form

$$\bar{\theta} = \theta_{QCD} + 6\varphi(x), \quad (12)$$

where the nondynamical phases in the quark mass matrices and the phase of the complex constant k_3 can be included in

θ_{QCD} by a chiral rotation. In close analogy with the KSVZ scenario [12], our $\varphi(x)$ also receives a potential from the instanton background so as to develop a vacuum expectation value which enforces $\bar{\theta} \equiv 0$ [i.e., $\langle \varphi \rangle = -\theta_{QCD}/6$], to all orders in perturbation theory. Rather than the quarks, it is thus the gluino which realizes the Peccei-Quinn mechanism of solving the strong CP problem.

The axion, $a \equiv v_s[\varphi(x) - \langle \varphi \rangle]$, has a mass and lifetime given by

$$m_a \sim m_\pi \frac{f_\pi}{f_a}, \quad \tau(a \rightarrow 2\gamma) \sim \left(\frac{m_\pi}{m_a}\right)^5 \tau(\pi \rightarrow 2\gamma), \quad (13)$$

where its decay constant f_a is equal to $v_s/6$. Our axion is not a DFSZ axion [11] as it does not couple to quarks and leptons; it is also not a KSVZ axion [12] as it does not couple to unknown colored fermion multiplets beyond the MSSM spectrum. We may call it the *gluino axion* [16] as it is induced by promoting the masses of the gauginos to local operators.

Let us choose $v_s/\sqrt{2} \sim 10^{11}$ GeV, which is in the middle of the range of 10^9 to 10^{12} GeV allowed by astrophysical and cosmological bounds [17] on f_a . The effective theory below v_s is then a replica of the MSSM with the effective μ parameter

$$\mu_{eff} = \frac{v_s^2}{2M_{Pl}} e^{-i\theta_{QCD}/3} \sim 10^3 \text{ GeV} \times e^{-i\theta_{QCD}/3}, \quad (14)$$

which is the right scale for supersymmetry breaking. This seesaw mechanism for the μ parameter results from the introduction of the composite operator given by Eq. (6) into the theory, the dynamics of which are presumably dictated by physics at or near the Planck scale. On the other hand, the scale of the μ parameter is fixed by the astrophysical and cosmological bounds on the axion decay constant f_a which gives (or receives) a meaning to (from) the intermediate scale v_s [18].

The low-energy effective theory is the softly broken MSSM with R parity conservation. Indeed, after replacing the effective μ parameter [Eq. (14)] and $\langle \varphi \rangle = -\theta_{QCD}/6$ into the effective Lagrangian [Eq. (8)], we obtain

$$\begin{aligned} \mathcal{L}_{MSSM}^{soft} = & \tilde{Q}^\dagger M_Q^2 \tilde{Q} + \tilde{u}^{c\dagger} M_u^2 \tilde{u}^c + \tilde{d}^{c\dagger} M_d^2 \tilde{d}^c + \tilde{L}^\dagger M_L^2 \tilde{L} \\ & + \tilde{e}^{c\dagger} M_e^2 \tilde{e}^c + \{A_u \tilde{Q} \cdot H_u \tilde{u}^c + A_d \tilde{Q} \cdot H_d \tilde{d}^c \\ & + A_e \tilde{L} \cdot H_d \tilde{e}^c + \text{H.c.}\} + M_{H_u}^2 |H_u|^2 + M_{H_d}^2 |H_d|^2 \\ & + (\mu_{eff} B H_u \cdot H_d + \text{H.c.}) + \{M_3 \tilde{\lambda}_3^a \tilde{\lambda}_3^a + M_2 \tilde{\lambda}_2^i \tilde{\lambda}_2^i \\ & + M_1 \tilde{\lambda}_1 \tilde{\lambda}_1 + \text{H.c.}\}, \end{aligned} \quad (15)$$

which is nothing but the soft supersymmetry-breaking part of the MSSM Lagrangian. It is in fact this part of the Lagrangian that possesses all sources of CP violation through the complex A parameters, the gaugino masses,

and μ_{eff} itself. The explicit expressions for the mass parameters in Eq. (15) read as follows. The gaugino masses are given by

$$M_3 = |k_3| \mu_{eff}^*, \quad M_2 = k_2 \mu_{eff}^*, \quad M_1 = k_1 \mu_{eff}^*, \quad (16)$$

which are not necessarily universal. The soft masses for the Higgs sector are given by

$$\begin{aligned} M_{H_u}^2 = & y_u |\mu_{eff}^2|, \quad M_{H_d}^2 = y_d |\mu_{eff}^2|, \\ \mu_{eff} B = & |\mu_{eff}|^2 \left(\frac{8m_s^2}{v_s^2} + k_\mu \right), \end{aligned} \quad (17)$$

and are responsible for electroweak symmetry breaking, with similar expressions for the mass-squared matrices of the sfermion fields. In particular, since m_s^2/v_s^2 is of order unity, the B parameter is also of the same scale. Finally the A parameters are given by

$$A_u = \mu_{eff}^* k_u, \quad A_d = \mu_{eff}^* k_d, \quad A_e = \mu_{eff}^* k_e, \quad (18)$$

which do not have to be proportional to h_u , h_d , and h_e of Eq. (7) as in the constrained MSSM.

As noted before, all mass scales of the MSSM Lagrangian [Eq. (15)] are fixed in terms of $|\mu_{eff}|$. More than this, the phase of μ_{eff} , i.e., $-\theta_{QCD}/3$, contributes universally to all mass parameters which are complex. However, the phases of the gaugino masses as well as those of the A and B terms also depend on the k parameters. Hence if the flavor structure of these matrices is not the same as those of the usual quarks and leptons, then the CP violation in flavor-changing processes is a powerful probe [19] into this sector of the effective theory. In the calculation of electric dipole moments due to supersymmetry [9], these CP phases can be considered as they are without worrying about whether there is an additional contribution from $\bar{\theta}$.

In conclusion, we have presented in the above a natural solution to one hierarchy problem, i.e., why $\bar{\theta}$ is so small (the strong CP problem) and related it to another hierarchy problem, i.e., why μ is 1 TeV and not 10^{16} GeV (the μ problem). The primary difference between our approach and previous other attempts [20] lies in the fact that the gaugino masses are specifically promoted here to local operators given by $\mu(\hat{S})$. Indeed, finite bare mass terms for the gauginos would automatically break the $U(1)_{PQ}$ symmetry, making it impossible for the relaxation of $\bar{\theta}$ to zero. As it is, $\langle \hat{S} \rangle$ serves two important purposes: its magnitude determines the scale of supersymmetry breaking and its phase solves the strong CP problem.

Let us summarize our proposal.

(i) We work in the framework of supersymmetry and identify $U(1)_{PQ}$ as $U(1)_R$ which contains the usual R parity as a discrete subgroup.

(ii) We require the supersymmetric Lagrangian and all supersymmetry breaking terms to be invariant under $U(1)_R$.

(iii) We implement this with the composite operator $\mu(\hat{S}) \equiv (\hat{S})^2 / M_{Pl}$ where \hat{S} is a singlet superfield having $R = +1$.

(iv) The spontaneous breaking of $U(1)_R$ generates an axion and relaxes the effective QCD vacuum angle $\bar{\theta}$ to zero, using the dynamical gluino phase, thus solving the strong CP problem.

(v) The existing astrophysical and cosmological bounds on the axion decay constant f_a implies a supersymmetry breaking scale of 1 TeV.

(vi) The effective Lagrangian at low energy is that of the MSSM with R parity conservation. The μ problem is solved once $\langle S \rangle \sim f_a$ is assumed.

(vii) Since $\bar{\theta} = 0$ in this consistent supersymmetric theory, electric dipole moments can be calculated unambiguously from the other explicit CP violating terms of the MSSM.

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